Level – 1	Daily Tutorial Sheet - 4

46. **(i)** $I=\int e^x\left(\frac{1-x}{1+x^2}\right)^2 dx=\int e^x\left(\frac{1+x^2-2x}{\left(1+x^2\right)^2}\right)dx=\int e^x\left(\frac{1}{1+x^2}-\frac{2x}{\left(1+x^2\right)^2}\right)dx=\frac{e^x}{1+x^2}+c$

(ii) $\int e^x\left(\frac{1-\sin x}{1-\cos x}\right)dx=\int e^x\left(\frac{\operatorname{cosec}^2 x/2}{2}-\cot\frac{x}{2}\right)dx=-e^x\cot\frac{x}{2}+c$

$\left[\because \int e^x\left(f(x)+f'(x)\right)dx=e^x f(x)+c\right]$

(iii) $\int\sqrt{x}\,(\log x)^2\,dx=(\log x)^2\times\frac{2}{3}x^{3/2}-\int 2\log x\times\frac{1}{x}\times\frac{2}{3}x^{3/2}\,dx$

$=\frac{2}{3}x^{3/2}(\log x)^2-\frac{4}{3}\left(\log x\times\frac{2}{3}x^{3/2}-\int\frac{1}{x}\times\frac{2}{3}x^{3/2}dx\right)$

$=\frac{2}{3}x^{3/2}(\log x)^2-\frac{8}{9}x^{3/2}\log x+\frac{16}{27}x^{3/2}+c$

(iv) Apply by - parts taking x^2 as first part.

(v) Apply by - parts taking $\log\left(\sqrt{1-x}+\sqrt{1+x}\right)$ as first part and dx as second part.

$\int\log\left(\sqrt{1-x}+\sqrt{1+x}\right)dx=x\log\left(\sqrt{1-x}+\sqrt{1+x}\right)+\frac{x}{2}\,\frac{-1}{2}\left[\frac{x}{2}\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}\frac{x}{2}\right]$

(vi) Apply by - parts taking $\log\left(\tan\frac{x}{2}\right)$ as first part. $\int\cos x\log\left(\tan\frac{x}{2}\right)dx=\sin x\log\left(\tan\frac{x}{2}\right)-x+c$

47. Put $\tan x=t^2\Rightarrow x=\tan^{-1}t^2\Rightarrow dx=\frac{2t\,dt}{1+t^4}\Rightarrow I=\int\left(t+\frac{1}{t}\right)\frac{2t\,dt}{1+t^4}=2\int\frac{t^2+1}{t^4+1}dt$, then

Divide N^r and D^r by t^2 and proceed.

48. Divide N^r and D^r by $\cos^2 x$ and then put $\tan x=t$

49. $\int\frac{\sec^2\theta\,d\theta}{(\tan\theta-2)(2\tan\theta+1)}$ (Divide N and D by $\cos^2\theta$)

Let $\tan\theta=t$ *and* then solve by partial fractions.

50. $\int\frac{dx}{p+q\tan x}=\int\frac{\cos x\,dx}{p\cos x+q\sin x}$. Express $\cos x=l\,(p\cos x+q\sin x)+m\,(-p\sin x+q\cos x)$.

51. $I=\int\frac{dx}{\sin x\,(\sin x+2\cos x)}=\int\frac{\sec^2 x\,dx}{\tan x\,(\tan x+2)}=\int\frac{dt}{t\,(t+2)}$. Use partial fractions.

52. $\int\frac{2dx}{2\cos x+3}-\int\frac{\sin x}{2\cos x+3}dx=2\int\frac{\sec^2\frac{x}{2}}{2\left(1-\tan^2\frac{x}{2}\right)+3\left(1+\tan^2\frac{x}{2}\right)}dx+\frac{1}{2}\ln|2\cos x+3|+c$

Substitute $\tan\frac{x}{2}=t$ in first integral and solve further.

53. $\displaystyle\int \frac{\sin x}{(\sin x - \cos x)}\,dx = \frac{1}{2}\int \frac{(\sin x - \cos x) + (\sin x + \cos x)}{(\sin x - \cos x)}\,dx = \frac{x}{2} + \frac{1}{2}\,\ell n\,|\sin x - \cos x| + c$

54. $\displaystyle I = \int \frac{a}{\sqrt{a^2 - x^2}}\,dx + \int \frac{xdx}{\sqrt{a^2 - x^2}} = a\,\sin^{-1}\frac{x}{a} - \sqrt{a^2 - x^2} + c.$

55. $\displaystyle\int x\sqrt{\frac{a^2 - x^2}{a^2 + x^2}}\,dx = \int \frac{x\left(a^2 - x^2\right)}{\sqrt{a^4 - x^4}}\,dx$

Let $x^2 = t \Rightarrow 2xdx = dt$

$$\frac{1}{2}\int \frac{\left(a^2 - t\right)}{\sqrt{a^4 - t^2}}\,dt = \frac{a^2}{2}\int \frac{dt}{\sqrt{\left(a^2\right)^2 - t^2}} - \frac{1}{2}\int \frac{tdt}{\sqrt{\left(a^2\right)^2 - t^2}}$$

$$= \frac{a^2}{2}\sin^{-1}\left(\frac{t}{a^2}\right) + \frac{1}{2}\sqrt{a^4 - t^2} = \frac{a^2}{2}\sin^{-1}\left(\frac{x^2}{a^2}\right) + \frac{1}{2}\sqrt{a^4 - x^4} + c$$

56. Put $x = \sin\theta \Rightarrow I = \displaystyle\int \frac{\theta\cos\theta\,d\theta}{\cos^3\theta} = \int \theta\sec^2\theta\,d\theta$. Apply by parts.

57. Apply by parts taking $\log(1 + x^2)$ as I part and dx as second part.

58. Apply by-parts taking $\log(\sec x + \tan x)$ as I part and $\sin x$ as second part.

59. $\displaystyle\int \frac{\cos(x - a + a)}{\cos(x - a)}\,dx = \int \frac{\cos(x - a)\cos a - \sin(x - a)\sin a}{\cos(x - a)}\,dx = \int\left[\cos a - \tan(x - a)\sin a\right]dx$

60. Put $x = \cos\theta \Rightarrow I = -\displaystyle\int \frac{\theta}{2}\sin\theta\,d\theta$. Apply by parts.